Graph Neural Networks for Traffic Forecasting

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17th INTERNATIONAL OPERATIONS & MAINTENANCE CONFERENCE IN THE ARAB COUNTRIES
OMAINTEC 2019 - 19 November
Dubai, United Arab Emirates

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Introduction

- Rapid urbanization: 4.7B people to 6.7B by 2050 [United Nations]
- Air and water pollution, unsustainable energy consumption, toxic waste disposal, inadequate urban planning, decreased public health and safety, social vulnerability
- In most large cities of the world, **mobility of passengers and freights is not yet sustainable**
- Traffic congestion, increase of transport needs, ineffective accessibility, reduced productivity

Introduction

- Rapid urbanization: 4.7B people to 6.7B by 2050 [United Nations]
- Air and water pollution, unsustainable energy consumption, toxic waste disposal, inadequate urban planning, decreased public health and safety, social vulnerability
- In most large cities of the world, **mobility of passengers and freights is not yet sustainable**
- Traffic congestion, increase of transport needs, ineffective accessibility, reduced productivity
- Solutions:
  - 1. Champion alternatives
  - 2. Enlarge infrastructure
  - **3. Manage traffic flows**
- Increase of available data enables innovative and integrated solutions
- Urban computing: intelligent transportation systems (ITS), smart vehicle sharing systems, home automation, smart grid and energy solutions
- Core component of ITS:
  - **traffic forecasting**

Traffic Forecasting

Goal of Traffic Forecasting:

- **Measure, model and predict traffic conditions,** in real-time, accurately and reliably, in order to
- **Optimize the flow and mitigate traffic congestion**, and support traffic light control, time of arrival estimates, planning of new road segments

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Very challenging problem:

- **Heterogeneous data** (e.g., loop counter and floating car data)
- Complex **spatio-temporal** dependencies
- Typically **sparse, incomplete and high-dimensional** data
- Inclusion of **external factors** (e.g., weather conditions, and road accidents)

Traffic Forecasting

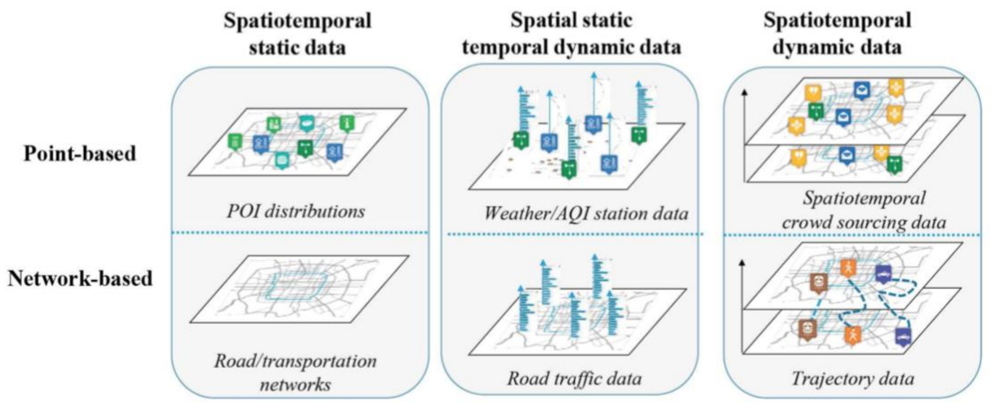


Figure 1: Six types of urban data [9]. Reprinted with permission.

Traffic Forecasting

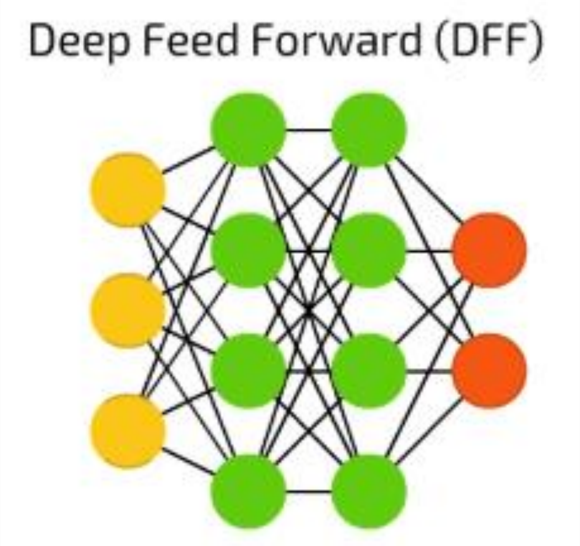
- Traditional approaches can be divided into model-driven and data-driven approaches

- **Model-driven** methods typically require prior knowledge, detailed modeling, not easily transferable to other cases, significant computational resources

- **Traditional data-driven** approaches typically require careful feature engineering, and are not complex enough to model the non-linearity and non-stationarity of the spatio-temporal data

Neural Networks

- Deep learning and neural networks



Source: Van Veen, F. & Leijnen, S. (2019). The Neural Network Zoo. Retrieved from https://www.asimovinstitute.org/neural-network-zoo

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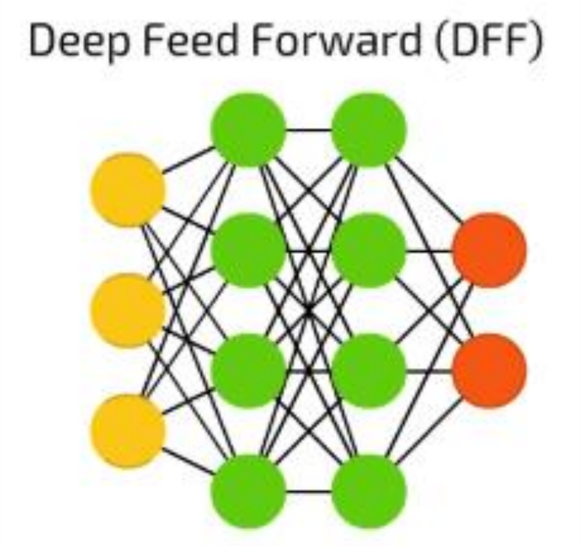

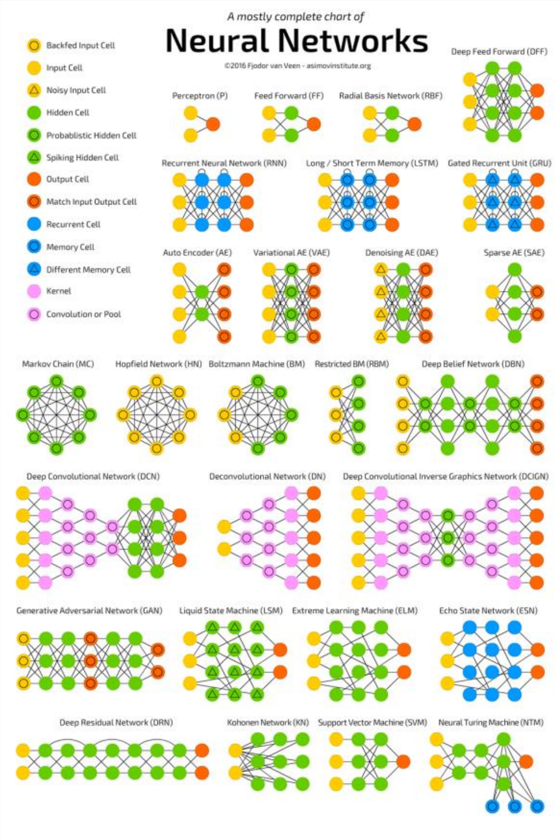

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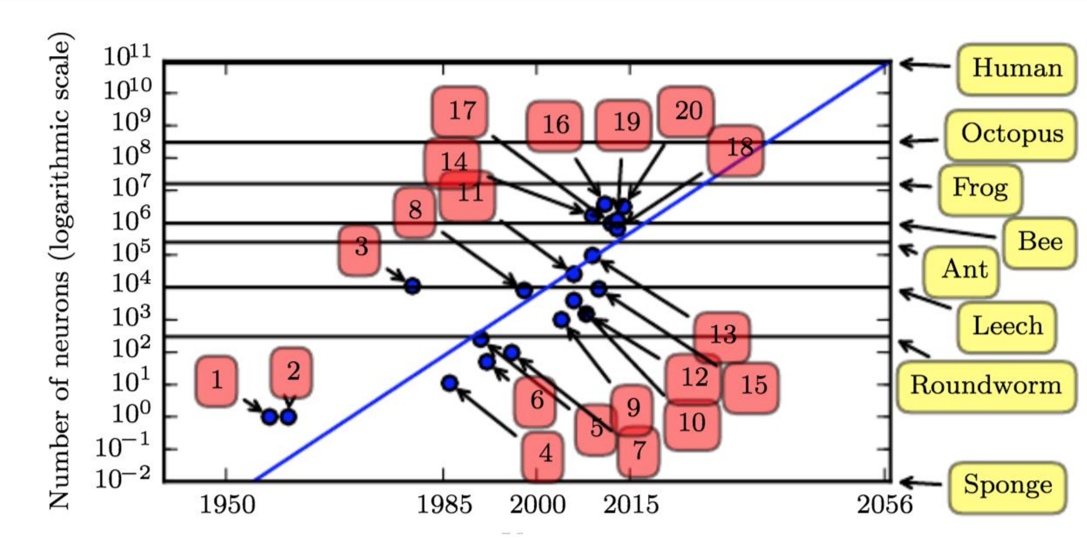
Neural Networks

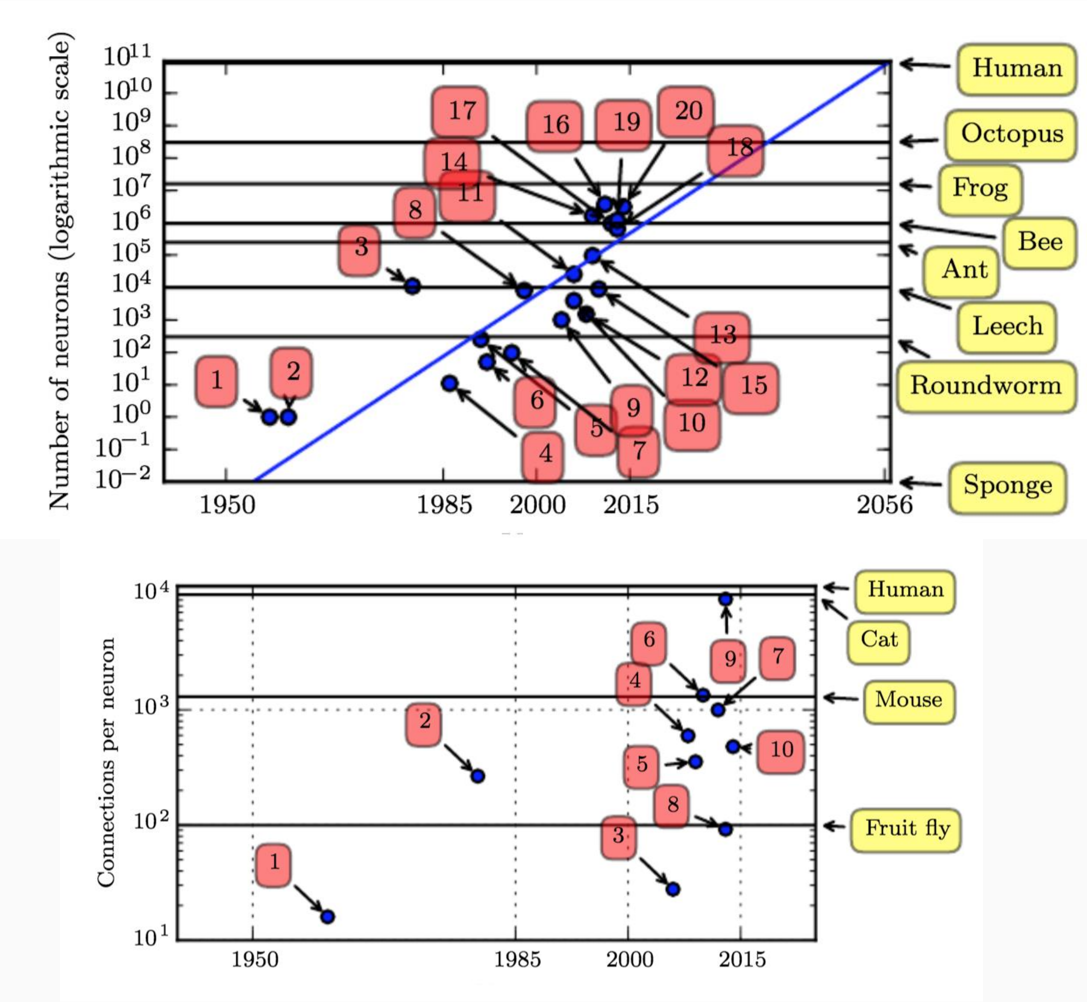
- # neurons



Source: ***Deep Learning*** - Ian Goodfellow et al, MIT PRESS - 2017

Neural Networks

- # neurons
- # connections



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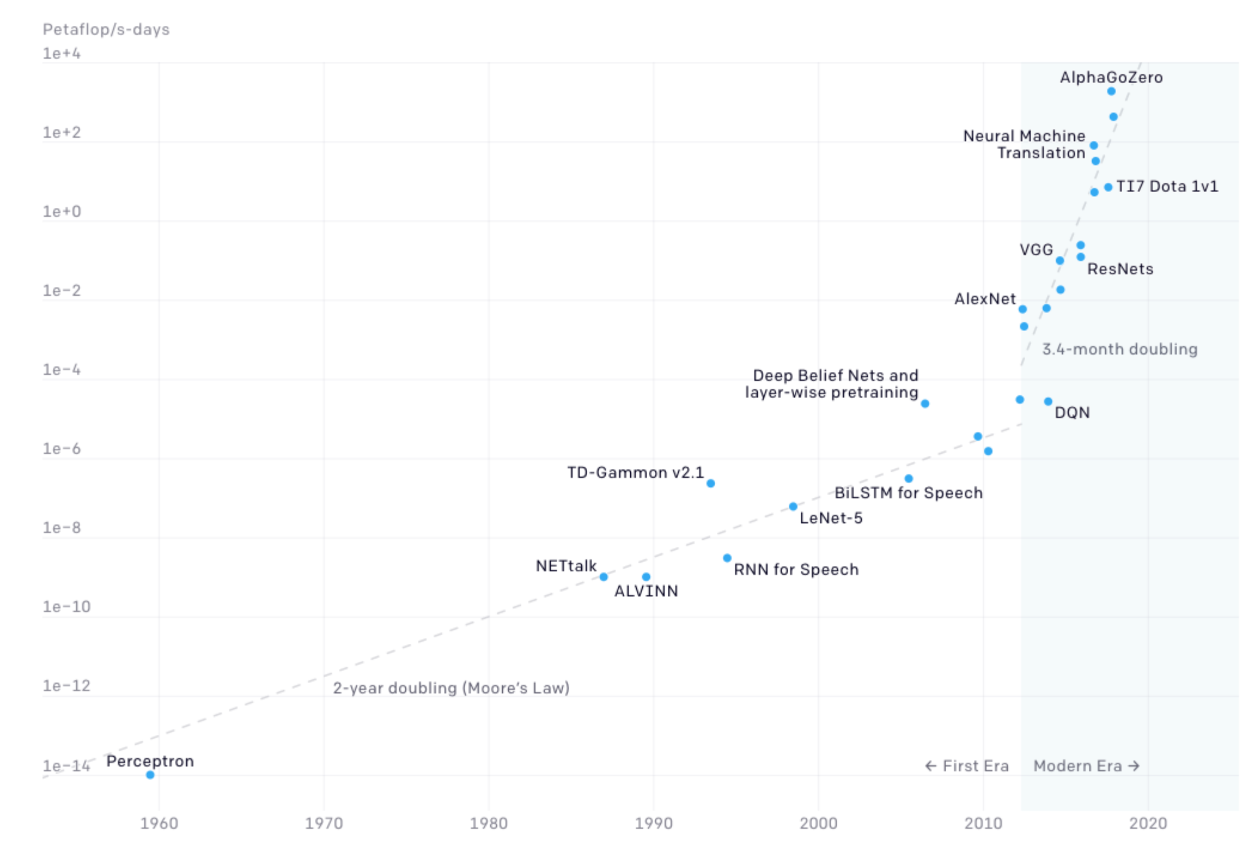
Neural networks (datasets and compute)



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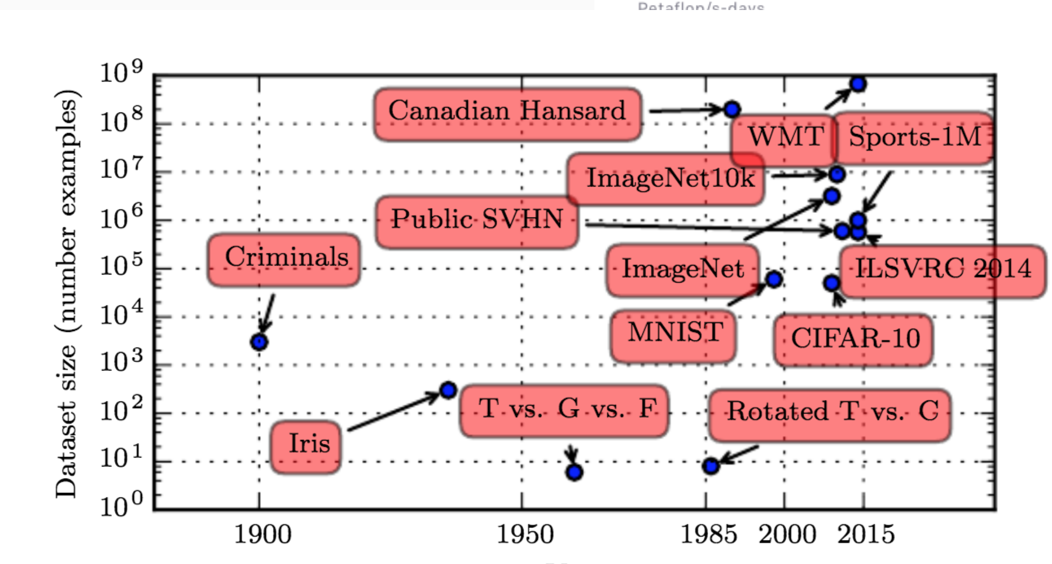

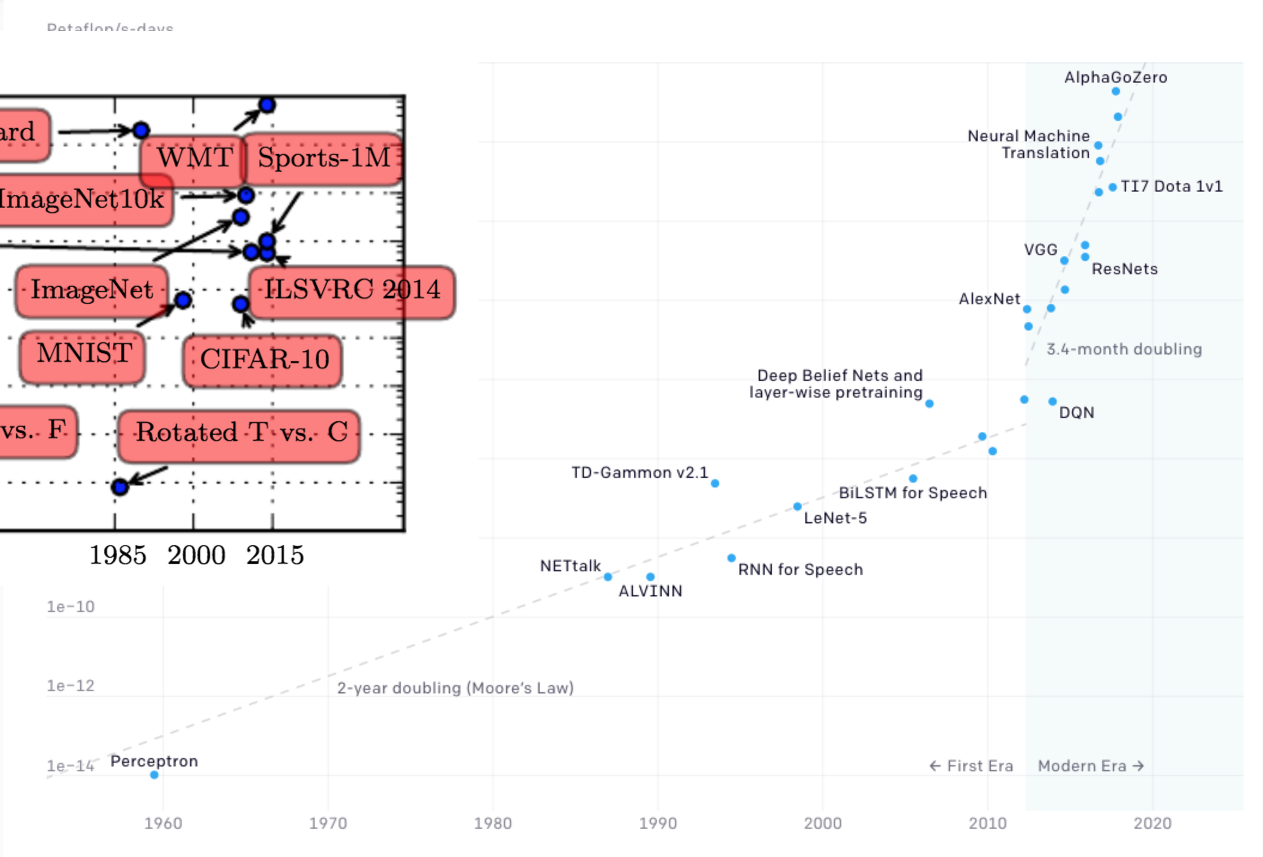

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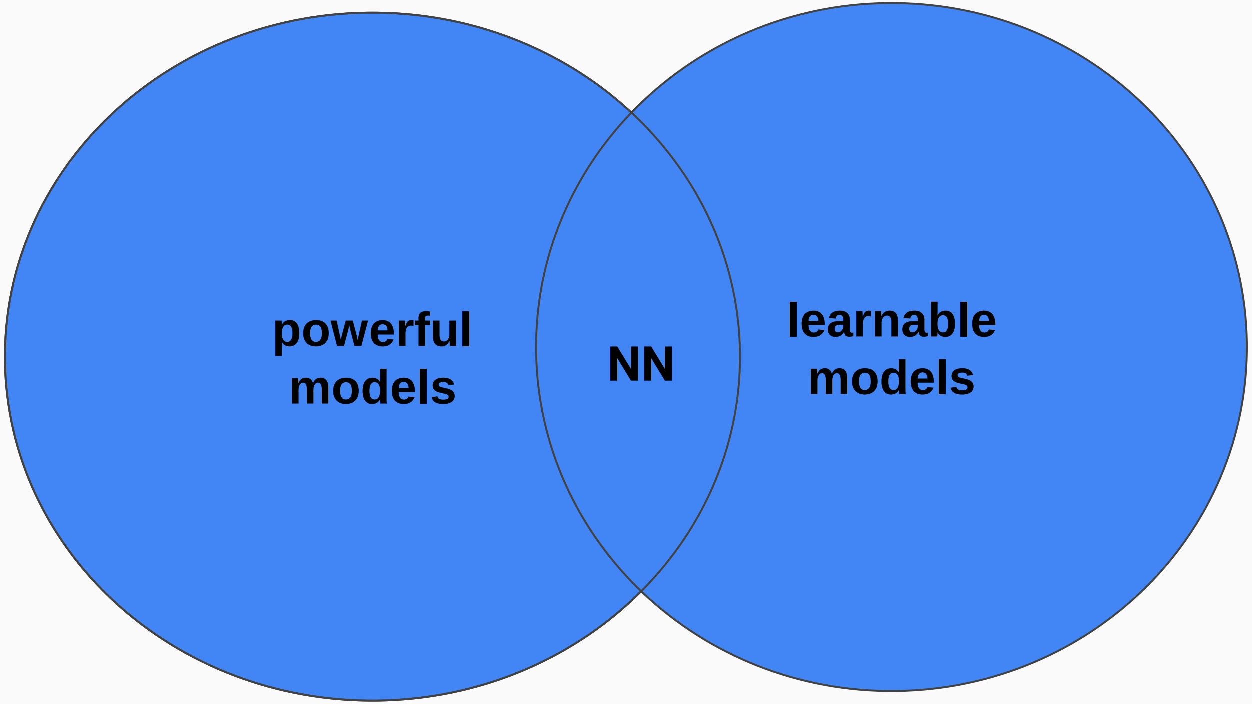
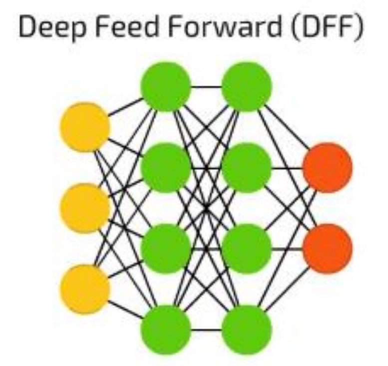


Convolutional Neural Networks (CNN)



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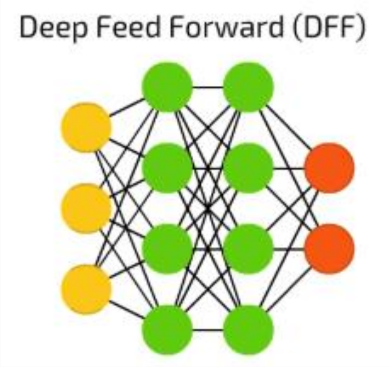

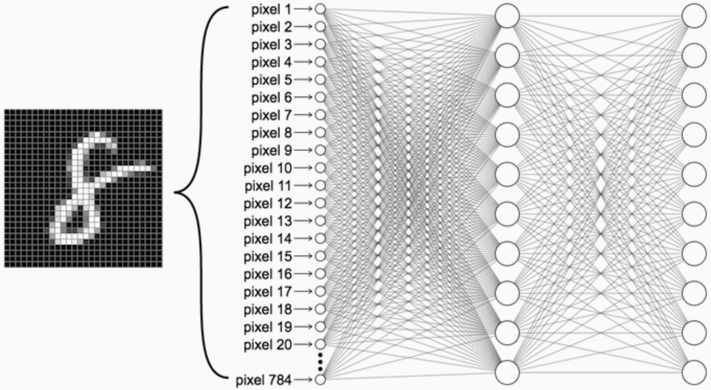

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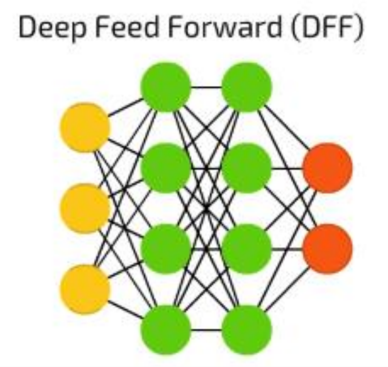

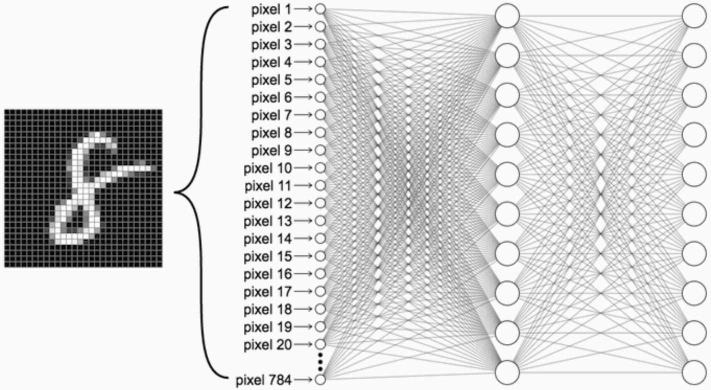

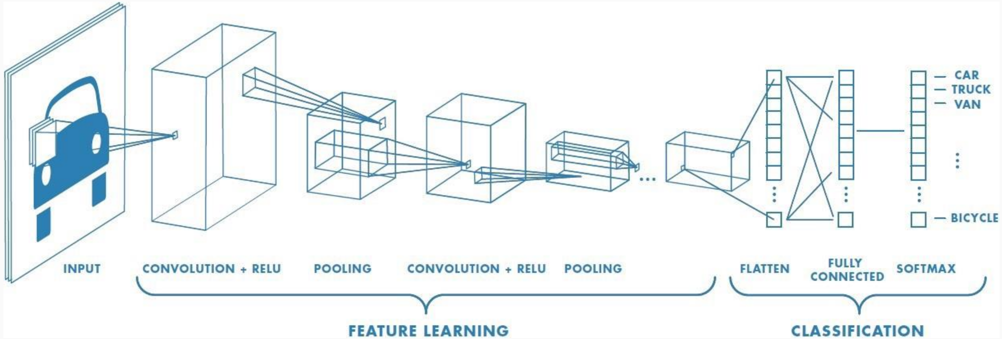

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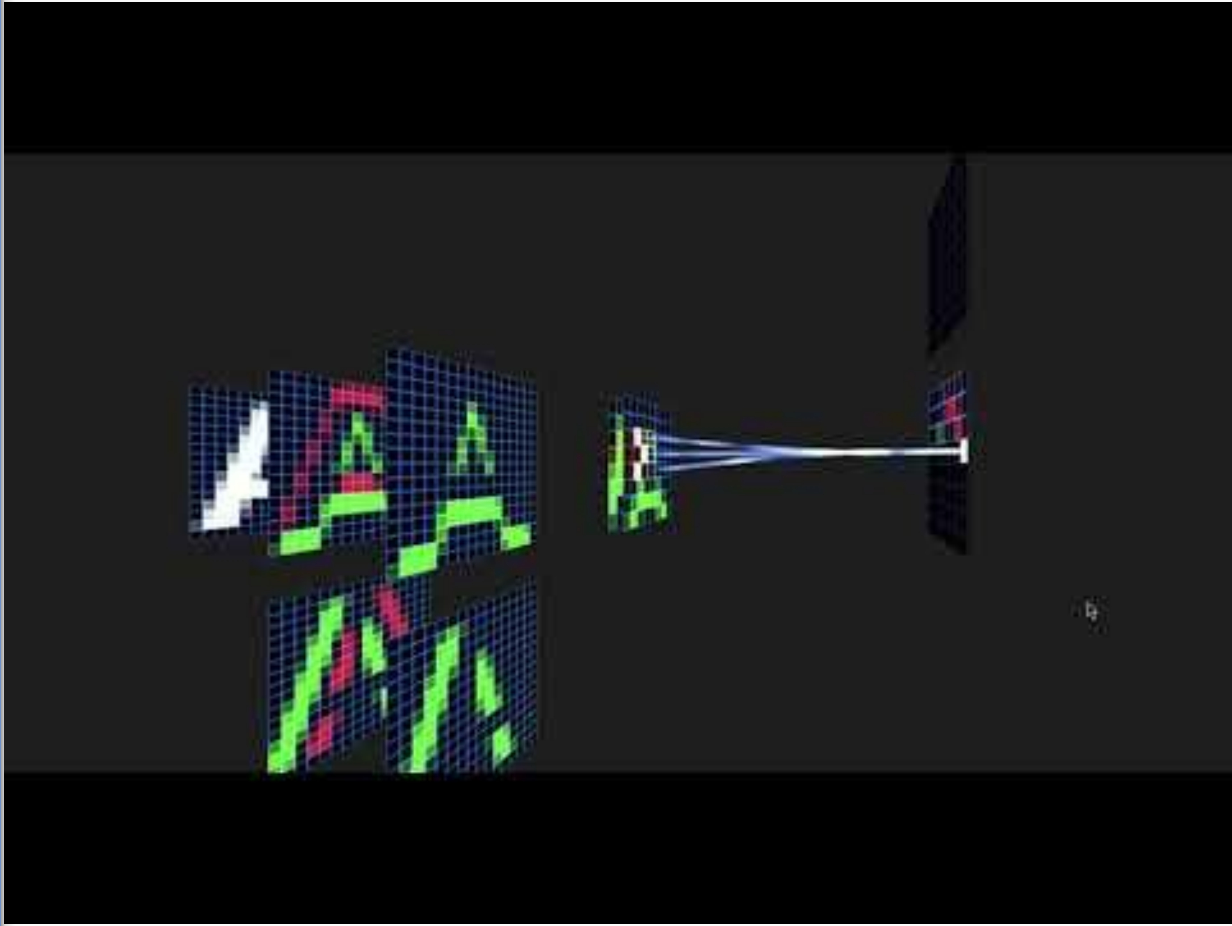

https://youtu.be/9KuhzUX1_Ks

Source: Otavio Good - A visual and intuitive understanding of deep learning

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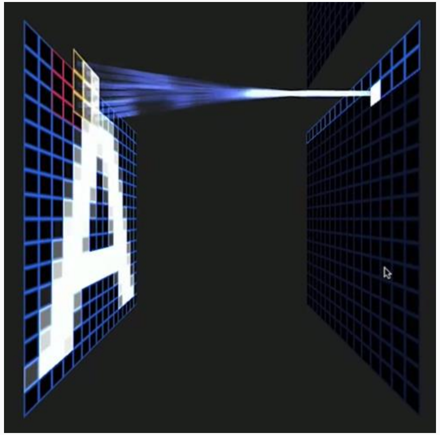

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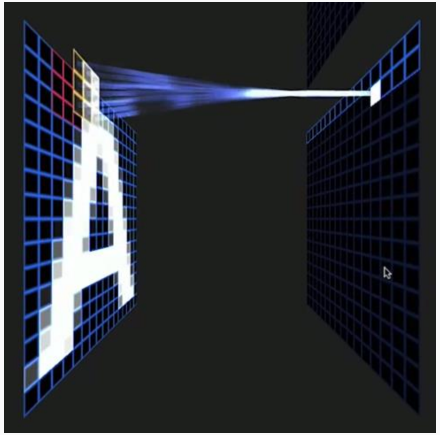
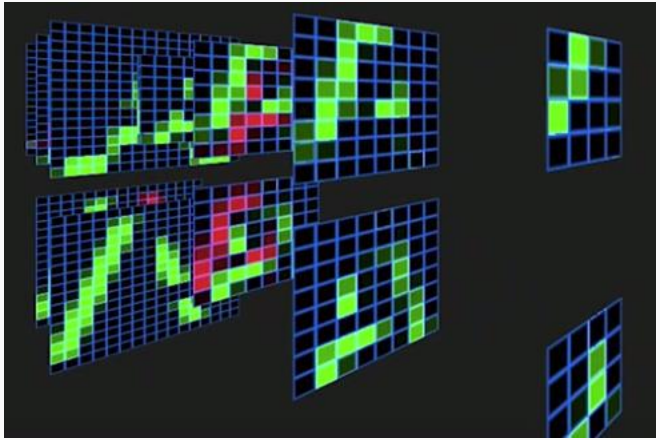
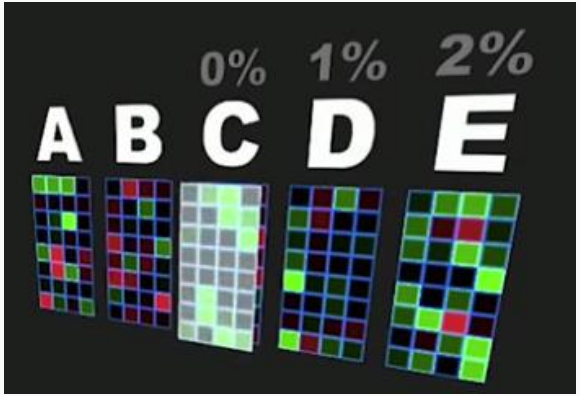
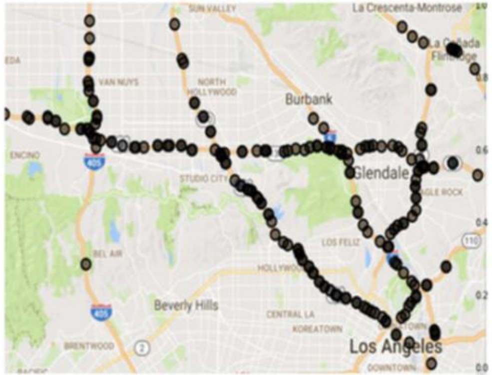


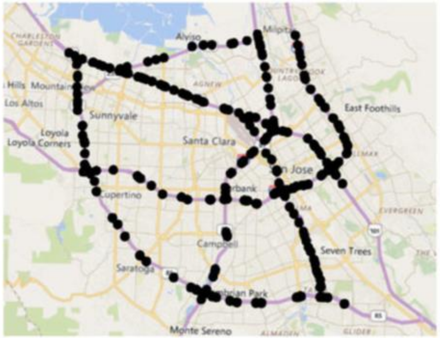




Graph Neural Networks (GNN)



(a) METR-LA



(b) PEMS-BAY

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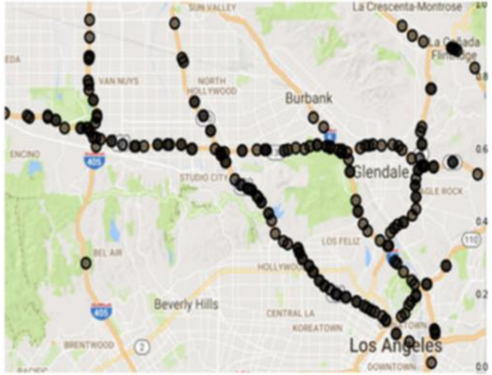

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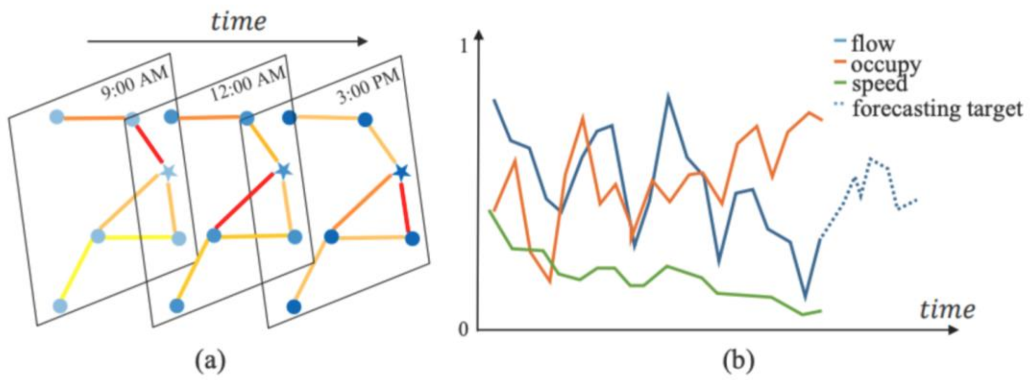

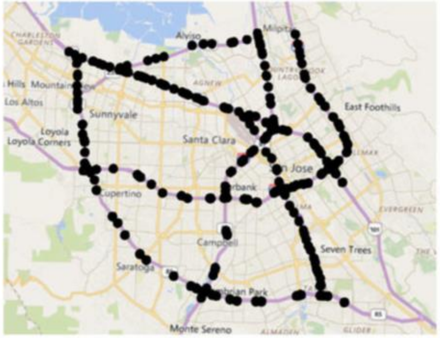

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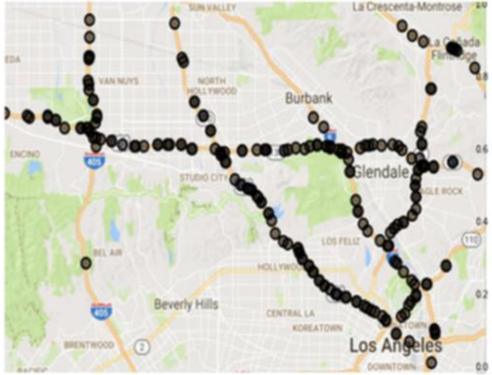

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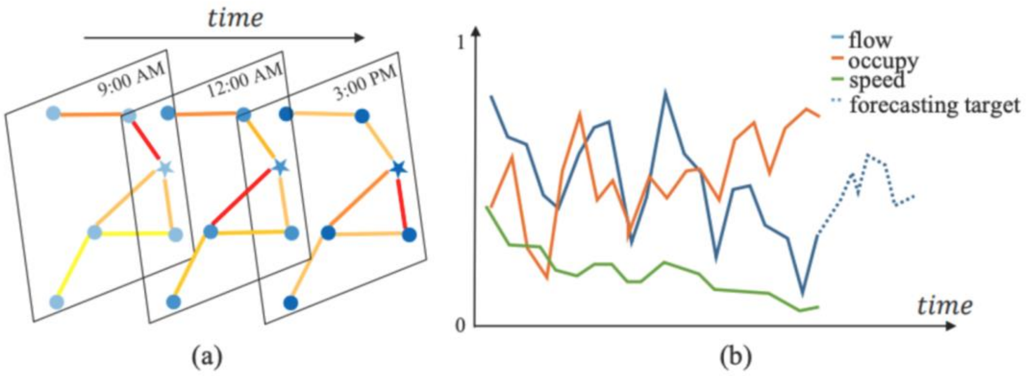

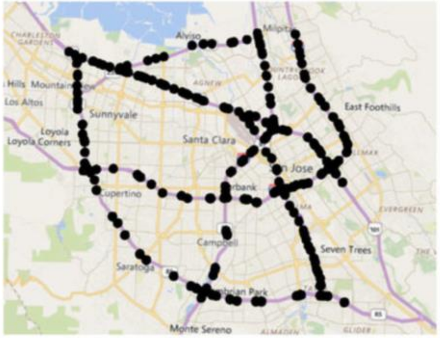

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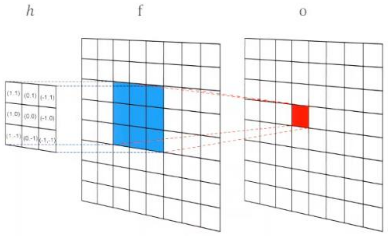
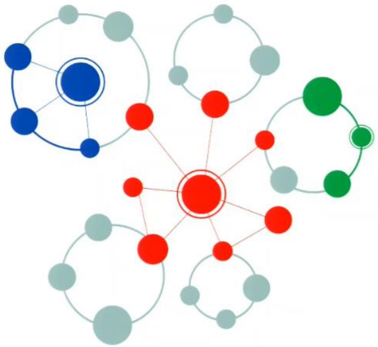


$$g(m,n) = (h * f)(m,n) = \sum_{i,j=-w}^{w} h(i,j) \cdot f(m-i, n-j)$$



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Category	References
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Convolutional Graph Neural Networks	[37], [40], [51]–[53]
Graph Attention Networks	[54], [55]
Graph Autoencoders	[56]–[61]

Table 1. Categorization of graph neural network models and representative publications.

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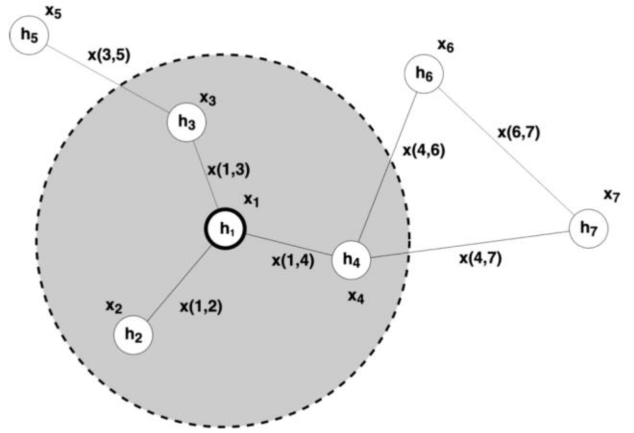

$$h_1 = (x_1, x_{(1,2)}, x_{(1,3)}, x_{(1,4)}, h_2, h_3, h_4, x_2, x_3, x_4)$$

$$h_v = f(x_v, x_{co[v]}, h_{ne[v]}, x_{ne[v]}),$$

$$\boldsymbol{o}_v = g(\boldsymbol{h}_v, x_v),$$

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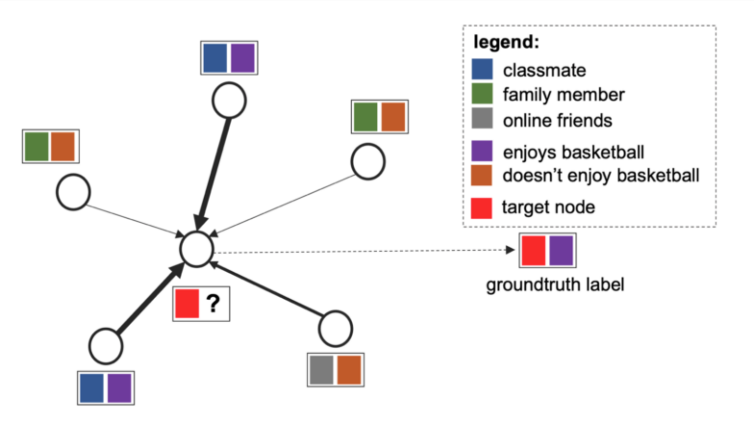

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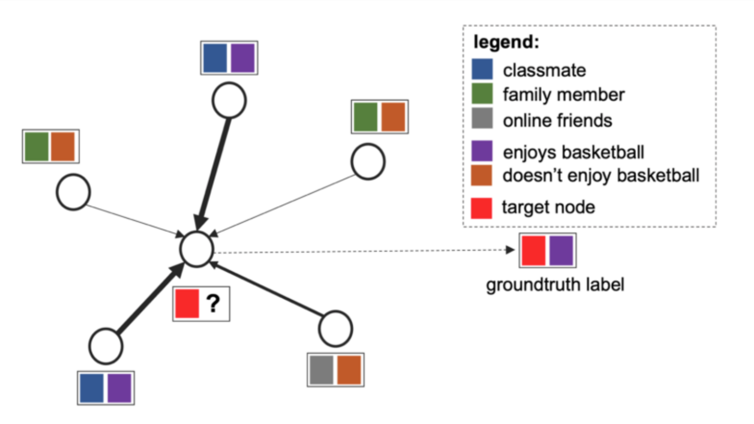

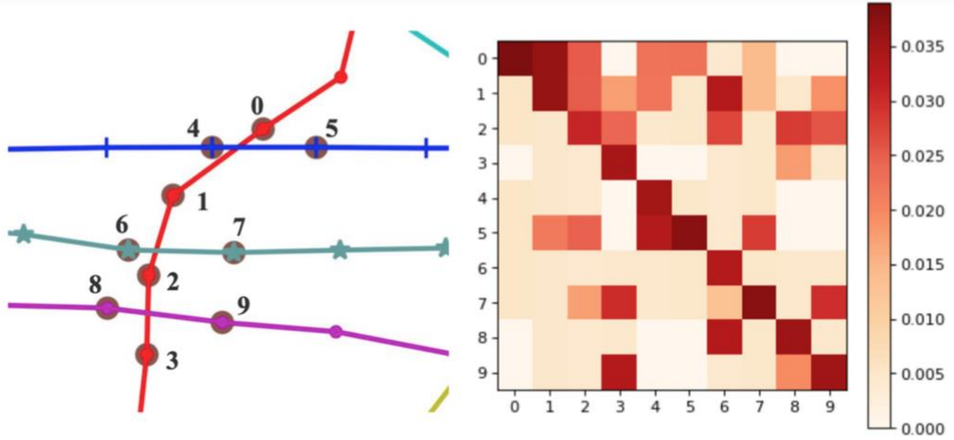

(c) Attention matrix obtained from the spatial attention mechanism.

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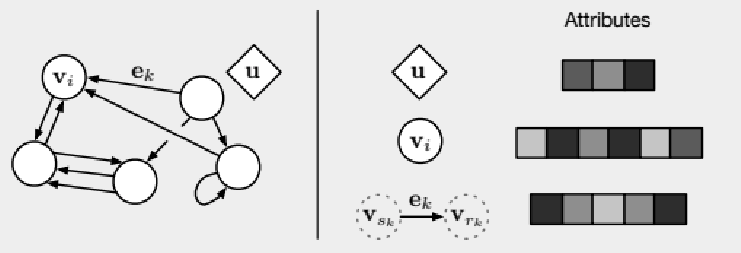
Graph Network (GN) Blocks



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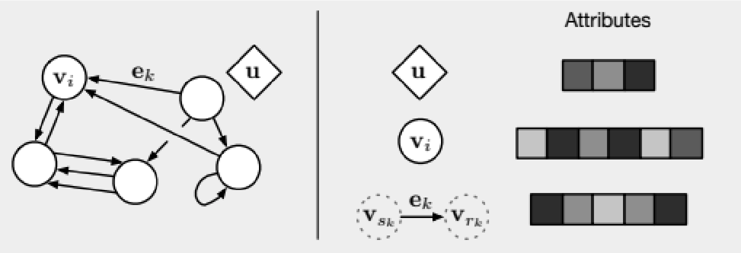

$$e_k' = \varphi^e(e_k, h_{rk}, h_{sk}, u), \qquad e_i^{*\prime} = \rho^{e\to h}(E_i'),$$

$$h_i' = \varphi^h(e_i^{*\prime}, h_i, u), \qquad e^{*\prime} = \rho^{e\to u}(E'),$$

$$u' = \varphi^u(e^{*\prime}, h^{*\prime}, u), \qquad h^{*\prime} = \rho^{h\to u}(H'),$$

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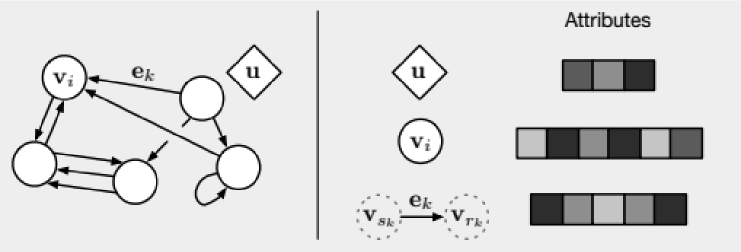

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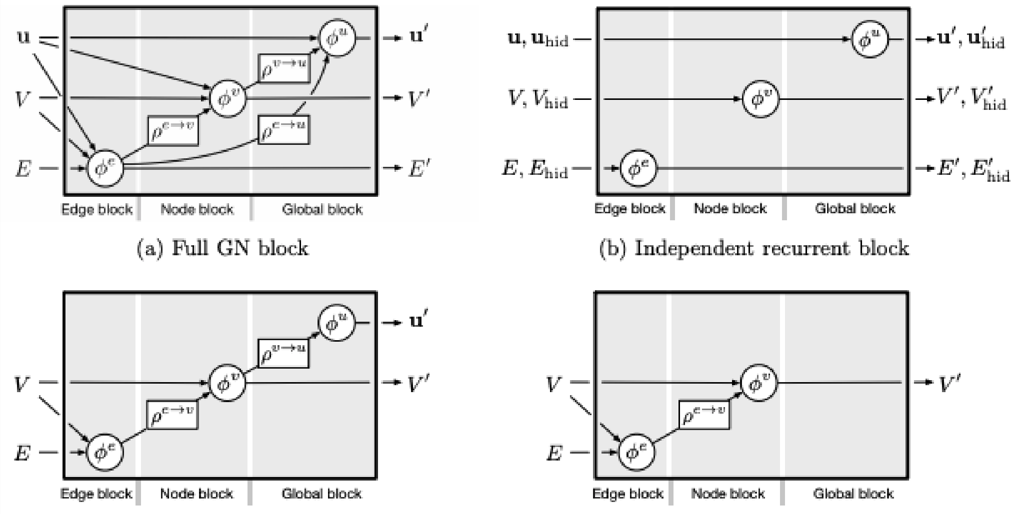

(a) Full GN block
(b) Independent recurrent block
(c) Message-passing neural network
(d) Non-local neural network

GNN for Traffic Forecasting

=

GNN

+

Time

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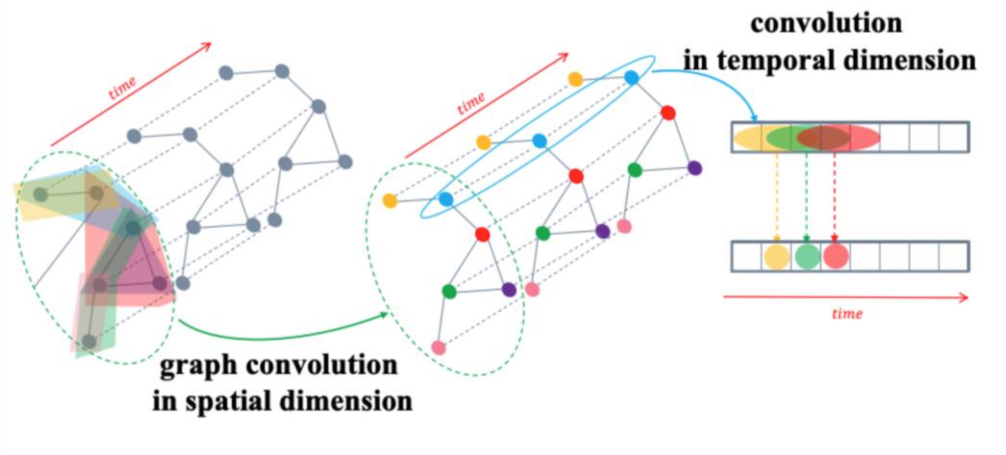

GNN for Traffic Forecasting

Model	Ref.	Scope	Predicts	Data source	Datasets	Open dataset?	Code available?
ST-GCN	[92]	Fw, Ur	S	L	BJER4, PeMS	✗, ✓	✓
DCRNN	[83]	Fw	S	L	METR-LA, PeMS	✓	✓
MRes-RGNN	[87]	Fw	S	L	METR-LA, PeMS	✓	✗
TGC-LSTM	[7]	Fw, Ur	S	L, FCD	LOOP, INRIX	✓, ✗	✗
ASTGCN	[8]	Fw	F, S	L	PeMSD4, PeMSD8	✓	✓
STDGI	[86]	Fw	S	L	METR-LA	✓	✓
MVGCN	[90]	Ur	F	FCD	TaxiNYC, TaxiBJ, BikeDC, BikeNYC	✓	✗
DST-GCNN	[82]	Fw, Ur	S, V	L, FCD	METR-LA, TaxiBJ	✓	✗
GSRNN	[91]	Ur	F	FCD	BikeNYC, TaxiBJ	✓	✗
Graph Wavenet	[84]	Fw	S	L	METR-LA, PeMS	✓	✓
3D-TGCN	[6]	Fw	S	L	PeMS	✓	✗
ST-UNet	[93]	Fw	S	L	METR-LA, PeMS	✓	✗
GaAN	[55]	Fw	S	L	METR-LA	✓	✗
Motif-GCRNN	[88]	Ur	S	FCD	TaxiChengdu	✗	✗
STGi-ResNet	[85]	Ur	F	FCD	Didi Chengdu	✓	✗
T-GCN	[94]	Fw, Ur	S	FCD	SZ-taxi, Los-loop	✗, ✓	✗
FlowConvGRU	[97]	Ur	F	FCD	TaxiNYC, TaxiCD	✓	✗

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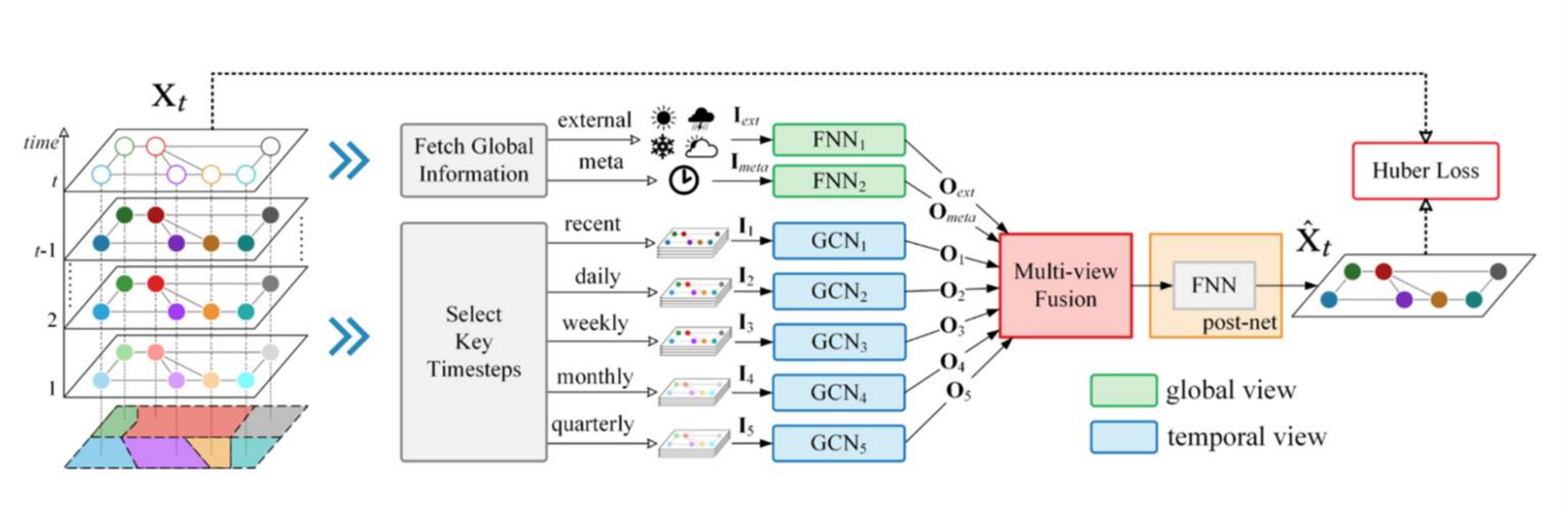

Multi-view Graph Convolutional Networks (MVGCN) [90]

Conclusions, Challenges and Opportunities

- Deep learning and in particular, Graph Neural Networks, have achieved state of the art results in prediction tasks, including traffic prediction
- Several challenges and opportunities lie ahead, in the next decades

- Opportunities:
  - More and better data
  - More computational resources
- Challenges:
  - Uncertainty estimates
  - Interpretability
  - Integration with downstream applications
  - Data ageing and concept drift
  - Travel time prediction
  - Better evaluation metrics
  - Systematic inclusion of exogenous factors

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